SOLUTIONS TO

Written Exam at the Department of Economics summer 2020

Economics of the Environment and Climate Change

Final Exam

June 4, 2020

(3-hour open book exam)

Answers only in English.

This exam question consists of 4 pages in total, including this front page.

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Written exam in the Economics of the Environment and Climate Change, Spring 2020

OPTIMAL CLIMATE POLICY

In the following you will be asked to analyse the optimal climate policy using a highly simplified model of the interaction between the economy and the climate system. When you have carried out the formal analysis, you are invited to discuss the limitations of the model.

The timeline in the model is divided into two periods which can be thought of the as "the present" and "the future". We will use the following notation:

$C_i =$ consumption in period *i*, *i* = 1,2
$K =$ investment in period 1 = capital stock in period 2
$E =$ emission of CO_2 in period 1
$A =$ investment in abatement of CO_2 emissions in period 1
$D =$ damage cost of climate change in period 2
$u(C_i) =$ utility from consumption in period *i*, *i* = 1,2
$U =$ lifetime utility of the representative consumer
$Y =$ output in period 1 (exogenous)
$r =$ real rate of return on capital (exogenous)
$\rho =$ utility discount rate (exogenous)

The lifetime utility of the representative consumer is

$$U = u(C_1) + \frac{u(C_2)}{1+\rho}, \qquad u'(C_i) > 0, \qquad u''(C_i) < 0, \qquad i = 1,2. \tag{1}$$

At the beginning of period 1, the economy is endowed with a predetermined capital stock which generates an amount of output *Y* in period 1. During period 1 the existing capital stock is fully worn out, but a part of period 1 output can be accumulated as a new capital stock for use in period 2. Another part of period 1 output can be invested in abatement equipment *A* which can be used to reduce CO_2 emissions in period 1. Hence the amount of output left over for consumption in period 1 is

$$C_1 = Y - K - A. \tag{2}$$

In period 2, the capital accumulated during period 1 generates an amount of output $(1+r)K$, but the CO_2 accumulated in the atmosphere during period 1 leads to climate change which causes an output loss *D* in period 2, so consumption in that period is

$$C_2 = (1+r)K - D. \tag{3}$$

The damage cost in period 2 due to emissions in period 1 is

$$D = \beta E, \qquad \beta > 0, \tag{4}$$

where β is a constant. Emissions in period 1 vary positively with output but can be mitigated through abatement effort. Hence emissions are given by the following function where *a*, *b*, and θ are constant parameters:

$$E = aY - \frac{b}{\theta}A^{\theta}, \qquad a > 0, \quad b > 0, \quad 0 < \theta < 1. \tag{5}$$

The society considered can increase future consumption and welfare in two ways: it can invest in man-made capital, *K*, or it can invest in "natural capital" by undertaking abatement, *A*, which reduces the future damage cost of climate change. Both forms of investment involve some sacrifice of current consumption, as indicated by (2). The socially optimal investment policy is the combination of *K* and *A* that maximizes the lifetime utility (1) of the representative consumer.

Question 1. Use equations (2) through (5) to write the lifetime utility function (1) in terms of *K* and *A*.

Answer to Question 1. Inserting (4) and (5) in (3), we get

$$C_2 = (1+r)K - \beta aY + \frac{\beta b}{\theta}A^{\theta}. \tag{i}$$

Substituting (i) and (2) into (1), we can write lifetime utility as the following function of *K* and *A*:

$$U = u(Y-K-A) + \frac{u\left((1+r)K - \beta aY + \frac{\beta b}{\theta}A^{\theta}\right)}{1+\rho}. \tag{ii}$$

(End of answer to Question 1).

Question 2. Use your result in Question 1 to show that society's optimal investment policy implies that

$$\beta b A^{\theta-1} = 1+r. \tag{6}$$

Give an economic interpretation of this result and explain the intuition behind it.

Answer to Question 2. The optimal investment policy maximizes lifetime utility (ii) with respect to *K* and *A*. The first-order conditions for the solution to this problem are

$$\partial U / \partial K = 0 \quad \Rightarrow \quad u'(C_1) = \left(\frac{1+r}{1+\rho}\right) u'(C_2), \tag{iii}$$

$$\partial U / \partial A = 0 \quad \Rightarrow \quad u'(C_1) = \left(\frac{\beta b A^{\theta-1}}{1+\rho}\right) u'(C_2). \tag{iv}$$

Equating the right-hand sides of (iii) and (iv), we immediately get (6). The right-hand side of (6) is the return to investment in man-made capital, since it measures the increase in future consumption made possible by investing an extra "euro" in man-made capital today. The left-hand side of (6) is the return to investment in "natural" capital, since it measures the increase in future consumption made possible by investing an extra "euro" in abating CO_2 emissions today. Specifically, $bA^{\theta-1} = -dE / dA$ is the cut in current emissions obtained by investing an extra euro in abatement, and $\beta b A^{\theta-1}$ is the resulting drop in the future damage cost of climate change. Thus the optimal investment rule (6) says that the marginal return to investment in man-made capital should equal the marginal return to investment in natural capital (abatement). *(End of answer to Question 2).*

Question 3. Solve equation (6) for the optimal abatement effort *A* and explain intuitively how the various parameters/exogenous variables in (6) affect the optimal abatement effort.

Answer to Question 3. Solving (6) for A, we get

$$A = \left(\frac{1+r}{\beta b}\right)^{\frac{1}{\theta-1}} = \left(\frac{\beta b}{1+r}\right)^{\frac{1}{1-\theta}}. \tag{v}$$

Noting from (5) that $0 < \theta < 1$, we see from (v) that the optimal abatement effort increases with the marginal damage cost from climate change, β, which is intuitive: the marginal benefit from pollution abatement increases with the marginal cost of climate change. From (5) we have $dE / dA = -bA^{\theta-1}$, so a higher value of *b* means that additional investment in abatement is more effective in reducing emissions. That explains why the optimal abatement effort in (6) also increases with *b*. On the other hand, a higher marginal return *r* on investment in man-made capital makes investment in this form of capital more attractive relative to investment in natural capital. A higher value of *r* therefore reduces the optimal investment in abatement, according to (6). Since (v) implies that

$$\frac{\partial A}{\partial \theta} = \ln\left(\frac{\beta b}{1+r}\right)\left(\frac{\beta b}{1+r}\right)^{\frac{1}{1-\theta}} \frac{1}{(1-\theta)^2},$$

the effect of the parameter θ on the optimal abatement effort is ambiguous, as it depends on whether the fraction $\beta b/(1+r)$ is larger or smaller than 1. There is no obvious intuition for the latter result. *(End of answer to Question 3).*

Question 4. Use (5) to derive the marginal abatement cost *MAC*, i.e., the cost of reducing emissions by an extra unit. Is the *MAC* constant, increasing or decreasing? Briefly explain the intuition.

Answer to Question 4. From (5) we have $-dE/dA = bA^{\theta-1}$. This expression measures the reduction of emissions obtained by investing one more euro in abatement. The marginal abatement cost is the inverse of this expression, $1/(-dE/dA) = -dA/dE$. Hence we have

$$MAC = \frac{1}{-(dE/dA)} = \frac{1}{bA^{\theta-1}} = \frac{A^{1-\theta}}{b}. \qquad \text{(vi)}$$

Since we have assumed in (5) that $0 < \theta < 1$, we see from (vi) that the marginal abatement cost is increasing with the level of abatement effort *A*. This is intuitive, since it is likely be become increasingly difficult to reduce emissions further, the larger the cut in emissions producers have already undertaken. *(End of answer to Question 4).*

Question 5. Derive an expression for the Social Cost of Carbon (*SCC*), defined as the welfare cost of emitting an extra unit of CO_2. (Hint: Derive the effect on lifetime utility of emitting an extra unit of CO_2 in period 1. Then divide by the marginal utility of consumption in period 1 to obtain a welfare measure expressed in units of current consumption and use the first-order condition for the optimal choice of *K*). Give a brief, intuitive explanation for your result.

Answer to Question 5. According to (3) and (4) the emission of an extra unit of CO_2 in period 1 will reduce consumption in period 2 by the amount β. From (1) it follows that the resulting fall in lifetime utility will be $\beta u'(C_2)/(1+\rho)$. To convert this utility loss into an equivalent loss of current consumption, we divide by the marginal utility of consumption in period 1 to get

$$SCC = \left(\frac{\beta}{1+\rho}\right)\frac{u'(C_2)}{u'(C_1)}. \qquad \text{(vii)}$$

From the first-order condition (iii) we see that on the economy's optimal investment path we have

$$\frac{u'(C_2)}{u'(C_1)} = \frac{1+\rho}{1+r}. \qquad \text{(viii)}$$

Inserting (viii) in (vii), we obtain

$$SCC = \frac{\beta}{1+r}. \tag{ix}$$

According to (ix) the Social Cost of Carbon is the present value of the future damage cost caused by emitting an extra unit of CO_2 today. *(End of answer to Question 5).*

Question 6. Show that the result in *Question 2* can be restated in terms of *MAC* and *SCC* and explain the intuition behind it.

Answer to Question 6. Using (vi) and (ix), we can rearrange equation (6) as follows:

$$\beta b A^{\theta-1} = 1+r \quad \Leftrightarrow \quad \frac{\beta}{1+r} = \frac{A^{1-\theta}}{b} \quad \Leftrightarrow \quad SCC = MAC. \tag{x}$$

Thus the investment rule (6) - which says that investment in man-made and in natural capital should yield the same marginal return - is equivalent to requiring that the marginal cost of abating CO_2 emissions should equal the Social Cost of Carbon. The latter condition is a standard condition for optimal environmental policy requiring that the marginal cost of abating pollution should equal the marginal benefit, where the marginal benefit of abatement is the marginal damage cost avoided.

Question 7. Now suppose that the economy considered is organized as a market economy where the representative firm's total pollution-related costs in period 1 are

$$TC = A + \tau E, \tag{7}$$

where τ is a carbon tax levied by the government. As part of its maximization of profits, the firm chooses the abatement effort that minimizes its total pollution-related costs *TC*, given that emissions are determined by (5). Derive an expression for the value of τ that will implement the socially optimal abatement effort. Explain the intuition for your result.

Answer to Question 7. Inserting (5) in (7), we get

$$TC = A + \tau\left(aY - \tfrac{b}{\theta}A^{\theta}\right). \tag{xi}$$

The first-order condition for minimization of pollution-related costs with respect to abatement effort is

$$\frac{dTC}{dA} = 0 \quad \Rightarrow \quad \tau b A^{\theta-1} = 1. \tag{xii}$$

Using (vi), we can rewrite (xii) as

$$\tau = \frac{A^{1-\theta}}{b} \quad \Leftrightarrow$$

$$\tau = MAC. \qquad \text{(xiii)}$$

Thus a cost-minimizing firm will investment in abatement up to the point where the marginal abatement cost equals the carbon tax rate. Combining (ix) and (xiii), we see that the government can implement the socially optimal abatement effort by setting

$$\tau = SCC = \frac{\beta}{1+r}, \qquad \text{(xiv)}$$

The intuition is that the government can fully internalize the climate change externality by levying a carbon tax equal to the marginal external damage cost of emissions. *(End of answer to Question 7).*

Question 8. Apart from dividing time into only two periods, the model above is of course highly simplified in many ways. Discuss some of the complications that are left out from the model but which real-world policy makers have to face when designing a rational climate policy.

Answer to Question 8. Many limitations of the model above can be discussed here. Some of the most important points are the following:

a) There is a very high degree of uncertainty relating to the future damage cost of climate change. For example, climate scientists have come up with different estimates of the probability distribution for the Climate Sensitivity parameter which measures the long-term increase in the global mean surface temperature in case of a doubling of the concentration of CO_2 in the atmosphere. Climate scientists are also uncertain about the level of CO_2 concentration where the climate system may pass "tipping points" where the climate undergoes irreversible and catastrophic change (e.g., melting of ice sheets at the poles, reversal of the Gulf Stream, massive release of methane from melting permafrost areas, die-back of tropical rain forests, etc.). All of this means that there is a very large uncertainty regarding the size of the damage cost parameter β in the model above.

b) There is also considerable controversy over the size of the discount rate r that should be applied when calculating the Social Cost of Carbon. In an economy obeying the first-order condition (iii), which is the Ramsey Rule for an optimal intertemporal allocation of consumption, one can show that

$$r \approx \rho + \eta g,$$

where η is the numerical elasticity of the marginal utility of consumption, and g is the growth rate of consumption. Many scholars, including Nicholas Stern, have argued that the choice of the parameters ρ (determining the weight put on the welfare of future generations) and η (determining society's aversion to inequality of consumption) are ethical choices in a social cost-benefit analysis so that the discount rate should not necessarily correspond to the observed market return to capital. Because CO_2 emissions today have consequences for centuries from now, the choice of discount rate is crucial for the magnitude of the Social Cost of Carbon.

c) Adding to the difficulty of choosing an appropriate discount rate is the fact that there is uncertainty about the future marginal productivity of capital and hence about the level of future interest rates. It can be shown that such uncertainty implies that the discount rate applied in a social cost-benefit analysis of optimal climate policy should be lower the further into the future the benefits and costs occur. Hence uncertainty about future interest rates will in itself increase the SCC.

d) A simplified model like the one above may also underestimate the damage cost from climate change if the calibration of the parameter β does not account for the fact that part of the damage from climate change will take the form of destruction of environmental goods (ecosystems) and that such goods are likely to become increasingly scarce over time due to economic growth, even in the absence of climate change. Thus consumers are likely to place a relatively higher value on environmental goods in the future.

e) Given the large uncertainties mentioned above, it is very difficult to estimate the "correct" value of the SCC. The risk of irreversible catastrophic damages occurring when unknown tipping points are passed can motivate use of the precautionary principle of environmental policy where policy makers seek to keep global warming below a threshold where the risk of dangerous climate change is deemed to be small. This would imply that accumulated emissions should stay within a global "carbon budget". A carbon tax would be a cost-effective instrument of implementing the emissions reductions needed to respect the carbon budget.